AP Calculus BC

Polar Area

1) $r = 4 - 6\sin\theta$

$4 - 6\sin\theta = 0$

$-6\sin\theta = -4$

$\sin\theta = \frac{2}{3}$

$\theta = 0.729 \rightarrow A$

$\theta = 2.411 \rightarrow B$

$\text{Area} = \frac{1}{2}\int_A^B (4-6\sin\theta)^2 d\theta$

$= \underline{1.7635}$

2) $r = \cos 2\theta$

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

$\text{Area} = \int_0^{\pi/4} (\cos 2\theta)^2 d\theta$

$= \underline{0.3926}$

3) $r = 3 \quad r = 3\cos 3\theta$

$3\cos 3\theta = 0$

$\cos 3\theta = 0$

$3\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{6}$

$\text{Area} = \pi(3)^2 - 3\int_0^{\pi/6} (3\cos 3\theta)^2 d\theta$

$= \underline{21.2057}$

4) $r = 1 - \cos\theta$

$1 - \cos\theta = 1$

$\cos\theta = 0$

$\theta = \pi/2$

$\text{Area} = \frac{1}{2}\int_0^{\pi/2} (1-\cos\theta)^2 d\theta$

$= \underline{0.178}$

5) $r = 1 \quad r = 1 - \cos\theta$

$\text{Area} = \frac{1}{2}\pi - \int_0^{\pi/2} (1-\cos\theta)^2 d\theta$

$= \underline{1.2146}$

6) $r = 2 \quad r = 2 - 2\sin\theta$

$2 - 2\sin\theta = 0$

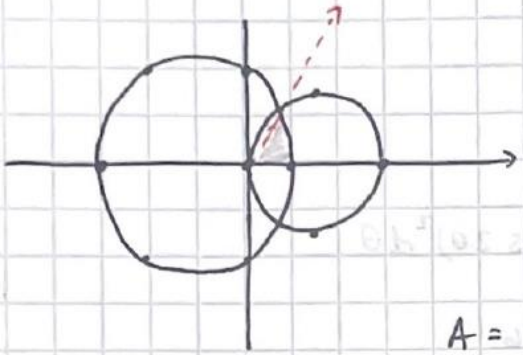
$\sin\theta = 1$

$\theta = \frac{\pi}{2}$

$\text{Area} = \int_0^{\pi/2} (2-2\sin\theta)^2 d\theta + \frac{1}{2}\pi(2)^2$

$= \underline{7.7079}$

7) $r = 3\cos\theta$ $\quad r = 2 - \cos\theta$



Find intersection:

$$3\cos\theta = 2 - \cos\theta$$
$$4\cos\theta = 2$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{3}$$

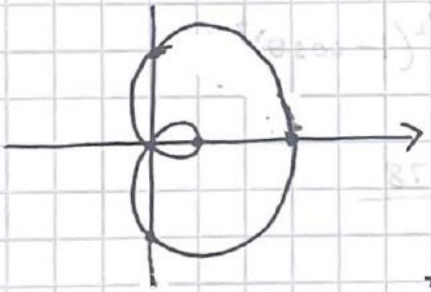
Polar Zero:

$$3\cos\theta = 0$$
$$\theta = \frac{\pi}{2}$$

$$A = \int_0^{\pi/3} (2-\cos\theta)^2 d\theta + \int_{\pi/3}^{\pi/2} (3\cos\theta)^2 d\theta$$

$$= 1.872$$

8) $r = 1 + 2\cos\theta$



$$1 + 2\cos\theta = 0$$
$$2\cos\theta = -1$$
$$\cos\theta = -\frac{1}{2}$$
$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\text{Area} = \int_0^{2\pi/3} (1+2\cos\theta)^2 d\theta - \frac{1}{2}\int_{2\pi/3}^{\frac{4\pi}{3}} (1+2\cos\theta)^2 d\theta$$

$$= 8.3377$$